Math 312 CAR **First Exam** **Date: 27/ 4 / 1434**

Name: **Co. No.** **Serial number**

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(QI) Mark true or false and justify your answer:

1. The intersection of open sets in a metric space X is an open set. [2]

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2. If A and B are convex subsets of $\mathbb{R}^n$, then $A \cup B$ is convex. [1]

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3. Every finite set in a metric space X is closed. [1.5]

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4. In $\mathbb{R}^n$ define $\|x\|$ by $\|x\| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$. Then $(\mathbb{R}^n, \|.\|)$ is a normed space. [2]

(QII) Complete the following: [3]

If (X, d) is a metric space and $A \subset X$, then

1. $x \in \ldots\ldots$ is a limit point of A if .....

2. $x \in \ldots\ldots$ is not an interior point of A if .....

3. A is an open set if .....

(QIII) Prove the following:

1. Compact subset of a metric space X is closed. [4]

2. If every infinite subset of S has a limit point in S, then S is closed. 2.5

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3. Let $f: S \to T$ be a function from a metric space (S, d_S) into a metric space (T, d_T) . Then f is continuous on S iff for every open set Y in T, $f^{-1}(Y)$ is open in S. 4